ON A CONJECTURE OF DOTSENKO ON WEAK NILPOTENCE AND THE ENGEL IDENTITY

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ABSTRACT. By means of AI-assisted computer algebra, we give examples of commutative but nonassociative algebras satisfying the identity $t_4 = 0$ such that neither the 7 nor 8-Engel identity holds. This resolves one conjecture of Dotsenko in the negative, and confirms a second. Conversely, we show that $t_4 = 0$ implies the 9-Engel identity. Together these show that 9 is the least integer m for which $t_4 = 0$ implies the m-Engel identity.

1. INTRODUCTION

Let A be a vector space over a field of characteristic zero, equipped with a commutative but not necessarily associative binary operation. One defines an n-linear operation $t_n(a_1,\ldots,a_n)$ as the sum of all distinct multilinear terms of degree n that one can define using said binary operation. For instance:

$$\begin{aligned} t_4(a,b,c,d) = {} & a[b[cd]] + a[[bc]d] + a[[bd]c] + [ab][cd] + [ac][bd] + [ad][bc] + [a[bd]]c \\ & + [a[bc]]d + [[ab]c]d + [[ac]b]d + [[ab]d]c + [[ad]b]c + [a[cd]]b + [[ac]d]b + [[ad]c]b. \end{aligned}$$

If t_i is identically 0 for all $i \geq n$, we say that A is *weakly nilpotent of index n*. We also define the *k-Engel* identity to be the following:

$$e_k(x,y) = \underbrace{[\cdots[x\,y]\cdots y]}_{k \text{ copies of } y} = 0,$$

ie., if it is identically 0 on an algebra A then k-fold iterated right multiplication by a fixed element y is always nilpotent of index k. Ternary variations of these conditions appear in Yagzhev's reformulation of the Jacobian conjecture, see [1, 5] for more details[1].

Dotsenko [2] considers the case where A is allowed to be non-commutative; using computer algebra he showed that that the non-commutative analogue[2] of the $t_4 = 0$ identity, $w_4 = 0$ implies the 7-Engel identity. Based on this intuition, he conjectures the following:

Conjecture 1.1. [2, Conjecture 5] Let A satisfy the $t_4 = 0$ identity. Then A also satisfies the 7-Engel identity ie., for all $x, y \in A$ one has $e_7(x,y) = 0$.

This conjecture is false. We phrase this as a theorem.

Theorem 1.2.

(1) *There exists a 17-dimensional algebra A satisfying the $t_4 = 0$ identity for which there exists $a, b \in A$ with $e_7(a,b) \neq 0$.*
(2) *There exists a 22-dimensional algebra B satisfying the $t_4 = 0$ identity for which there exists $a, b \in A$ with $e_8(a,b) \neq 0$.*

Using the same algebra A as in the theorem above, we confirm the following conjecture of Dotsenko.

Corollary 1.3. [2, Conjecture 4.1] *Suppose A satisfies the $t_4 = 0$ identity. It is not necessarily the case that $t_5 = 0$.*

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[1] This article was written some weeks before Alpöge announced a counterexample to the Jacobian conjecture in July 2026.
[2] note that in the non-commutative world there are more distinct multilinear terms of degree 4

The strategy behind both examples in Theorem 1.2 was to generate the free commutative, nonassociative algebra on two generators b_5, b_0 satisfying $t_4 = 0$ and compute quotients thereof to obtain small examples. We used Claude Code to write this implementation.

The same strategy can be used to produce the proof of the following which can be also verified by a relatively small program in Macaulay2.

Theorem 1.4.

(1) *Let A satisfy the $t_3 = 0$ identity. Then A also satisfies the identity $e_3 = 0$.*
(2) *Let A satisfy the $t_4 = 0$ identity. Then A also satisfies the identity $e_9 = 0$.*

The proof of both statements involves writing e_k as a sum. In the first case, this involves four terms and can be verified by hand while in the latter it involves 482 terms and requires computer algebra software to verify. In light of Theorem 1.2 the above theorem is sharp. This suggests the following modified conjecture. We remark that part (1) has previously been shown in [3].

Conjecture 1.5. Let A satisfy the $t_k = 0$ identity. Then A also satisfies the 3^{k-2}-Engel identity ie., for all $x, y \in A$ one has $e_{3^{k-2}}(x, y) = 0$.

We were unable to verify the $t_5 = 0$ version of this conjecture using the same method in a reasonable amount of time or memory, as the number of binary trees providing generators of the free algebra on two generators grows factorially and one would need to look at trees on 27 nodes, but we could can provide counterexamples similar to Theorem 1.2 for $k < 13$, showing that these do not suffice.

Finally, we make a comment on the methods of this paper. We believe the results illustrate that large language models can be useful coding assistants for practitioners in nonassociative algebra (and algebraic topology), enabling fast design of custom software for explicit computation of examples and counterexamples. We believe that this suits a particular genre of search-based problem, where auxiliary software used in the problem is permitted to be of arbitrary complexity and is not required to be correct, but the output can be checked by programs simple enough to be human-audited. For instance, in this case, the examples and derivation can be checked in the form of relatively short Macaulay2 files.

AI use disclosure: Claude Code (using the Opus 4.8 model) was used to produce all of the software used in the production of this document. It was not used in the writing of the paper itself other than for spell-checking purposes and the formatting of the expression in Appendix A.

2. The main results

In this section, we prove both theorems from the introduction.

Theorem 2.1. *There exists a 17-dimensional algebra A satisfying the $t_4 = 0$ identity for which there exists $a, b \in A$ with $e_7(a, b) \neq 0$.*

Proof. The following is the multiplication table for the counterexample. Let A be the commutative nonassociative $\mathbb{Q}$-algebra with basis $b_0, \ldots, b_{16}$ and commutative product $b_i b_j = b_j b_i$ given on the nonzero products by

$$\begin{array}{rcl@{\qquad}rcl@{\qquad}rcl}
b_0 b_0 &=& b_1 & b_0 b_{14} &=& -\frac{1}{4} b_{16} & b_2 b_5 &=& -2 b_8 - b_9 \\
b_0 b_1 &=& b_2 & b_0 b_{15} &=& \frac{1}{2} b_{16} & b_2 b_8 &=& b_{14} \\
b_0 b_2 &=& -\frac{1}{4} b_3 & b_1 b_1 &=& b_3 & b_2 b_9 &=& b_{15} \\
b_0 b_3 &=& -4 b_4 & b_1 b_2 &=& b_4 & b_2 b_{10} &=& b_{16} \\
b_0 b_5 &=& b_6 & b_1 b_6 &=& b_9 & b_2 b_{11} &=& -\frac{1}{2} b_{16} \\
b_0 b_6 &=& b_7 & b_1 b_7 &=& b_{11} & b_3 b_5 &=& -4 b_{10} - 8 b_{11} \\
b_0 b_7 &=& b_8 & b_1 b_8 &=& b_{12} & b_3 b_6 &=& -2 b_{13} \\
b_0 b_8 &=& -\frac{1}{2} b_{10} - \frac{1}{2} b_{11} & b_1 b_9 &=& b_{13} & b_3 b_7 &=& -2 b_{15} \\
b_0 b_9 &=& b_{10} & b_1 b_{10} &=& 4 b_{14} & b_3 b_8 &=& b_{16} \\
b_0 b_{10} &=& b_{12} & b_1 b_{11} &=& -2 b_{14} & b_4 b_5 &=& 4 b_{12} + b_{13} \\
b_0 b_{11} &=& -2 b_{12} - \frac{1}{2} b_{13} & b_1 b_{12} &=& \frac{1}{2} b_{16} & b_4 b_6 &=& 2 b_{14} \\
b_0 b_{12} &=& 2 b_{14} + \frac{1}{2} b_{15} & b_1 b_{13} &=& -2 b_{16} & b_4 b_7 &=& \frac{1}{2} b_{16} \\
b_0 b_{13} &=& -8 b_{14} - 2 b_{15} & & & & & &
\end{array}$$

all other products of basis vectors being 0 and extended bilinearly. The identity $t_4 = 0$ has been verified via a Macaulay2 script by showing it on basis elements. Since it holds on basis elements, it holds on the whole algebra by multilinearity.

Set $x = b_5$ and $y = b_0$. Then

$$e_7 = \underbrace{[\cdots[x\,y]\cdots y]}_{7 \text{ copies of } y} = \tfrac{1}{8}\,b_{16}$$

In particular the 7-Engel value $e_7(x, y) = \frac{1}{8}\,b_{16} \neq 0$, while $t_4(a, b, c, d) = 0$ for all $a, b, c, d \in A$. Thus this is a counterexample. □

Theorem 2.2. *There exists a 22-dimensional algebra B satisfying the $t_4 = 0$ identity for which there exists $a, b \in A$ with $e_8(a, b) \neq 0$.*

Proof. The $t_4 = 0$ and $e_8 \neq 0$ counterexample is produced similarly as in the previous theorem:

$$\begin{array}{rcl@{\qquad}rcl@{\qquad}rcl}
b_0 b_0 & = & b_1 & b_0 b_{19} & = & -\frac{1}{2}\,b_{20} & b_1 b_{19} & = & \frac{1}{2}\,b_{21} \\
b_0 b_1 & = & b_2 & b_0 b_{20} & = & \frac{1}{2}\,b_{21} & b_2 b_5 & = & -b_9 - 2\,b_{10} - b_{11} \\
b_0 b_2 & = & -\frac{1}{4}\,b_3 & b_1 b_1 & = & b_3 & b_2 b_9 & = & b_{17} \\
b_0 b_3 & = & -4\,b_4 & b_1 b_2 & = & b_4 & b_2 b_{10} & = & b_{18} \\
b_0 b_5 & = & b_6 & b_1 b_5 & = & b_7 & b_2 b_{11} & = & b_{19} \\
b_0 b_6 & = & b_8 & b_1 b_6 & = & b_{11} & b_2 b_{14} & = & -\frac{1}{2}\,b_{20} \\
b_0 b_7 & = & b_9 & b_1 b_7 & = & b_{13} & b_2 b_{16} & = & \frac{1}{4}\,b_{21} \\
b_0 b_8 & = & b_{10} & b_1 b_8 & = & b_{14} & b_3 b_5 & = & -4\,b_{12} - 4\,b_{13} - 8\,b_{14} \\
b_0 b_{10} & = & -\frac{1}{2}\,b_{12} - \frac{1}{2}\,b_{14} & b_1 b_9 & = & b_{15} & b_3 b_7 & = & 12\,b_{17} + 64\,b_{18} + 16\,b_{19} \\
b_0 b_{11} & = & b_{12} & b_1 b_{10} & = & b_{16} & b_3 b_8 & = & -6\,b_{17} - 32\,b_{18} - 10\,b_{19} \\
b_0 b_{12} & = & b_{16} & b_1 b_{12} & = & -b_{17} - 4\,b_{18} - 2\,b_{19} & b_3 b_{10} & = & b_{20} \\
b_0 b_{13} & = & -b_{15} & b_1 b_{13} & = & -4\,b_{17} - 16\,b_{18} - 4\,b_{19} & b_3 b_{12} & = & b_{21} \\
b_0 b_{14} & = & -2\,b_{16} & b_1 b_{14} & = & 3\,b_{17} + 14\,b_{18} + 4\,b_{19} & b_4 b_5 & = & b_{15} + 4\,b_{16} \\
b_0 b_{15} & = & -b_{17} & b_1 b_{16} & = & -\frac{1}{2}\,b_{20} & b_4 b_6 & = & -b_{17} - 6\,b_{18} - 2\,b_{19} \\
b_0 b_{16} & = & \frac{1}{2}\,b_{17} + 2\,b_{18} + \frac{1}{2}\,b_{19} & b_1 b_{17} & = & b_{21} & b_4 b_8 & = & \frac{1}{2}\,b_{20} \\
b_0 b_{17} & = & -b_{20} & b_1 b_{18} & = & -\frac{1}{4}\,b_{21} & b_4 b_{10} & = & -\frac{1}{4}\,b_{21} \\
b_0 b_{18} & = & \frac{1}{4}\,b_{20} & & & & & &
\end{array}$$

all other products of basis vectors being 0 and extended bilinearly. As above, the identity $t_4 = 0$ has been verified by a Macaulay2 script on basis elements, and holds on all of A by multilinearity.

Set $x = b_5$ and $y = b_0$. Then

$$e_8 = \underbrace{[\cdots[x\,y]\cdots y]}_{8 \text{ copies of } y} = -\tfrac{1}{16}\,b_{21}$$

In particular $e_8(x, y) = -\frac{1}{16}\,b_{21} \neq 0$, while $t_4(a, b, c, d) = 0$ for all $a, b, c, d \in A$ and so we can conclude. □

We remark that our second example alone is enough to refute [2, Conjecture 4.1].

Remark 2.3. These examples were generated by first constructing (a truncation of) the free commutative, non-associative algebra by the elements b_0 and b_5, We then construct a quotient of this algebra by the $t_4 = 0$ relation, and observed that the relations did not hold in the free algebra. We then reduced to produce the more tractable examples above.

We now resolve the second conjecture of Dotsenko.

Corollary 2.4. *Let A satisfy the $t_4 = 0$ identity. It is not necessarily the case that $t_5 = 0$.*

Proof. One computes $t_5(b_0, b_0, b_0, b_0, b_0) = 30 b_4$. □

Remark 2.5. An alternative proof runs as follows; in [4], just before the proof of Theorem 5, it is shown that if $t_4 = t_5 = 0$ in A then the 5-Engel identity holds. As the 5-Engel identity does not hold for A, it is not possible for the t_5 identity to hold.

Next, we verify the second main result of this paper:

Theorem 2.6.

(1) Let A satisfy the $t_3 = 0$ identity. Then A also satisfies the identity $e_3 = 0$.
(2) Let A satisfy the $t_4 = 0$ identity. Then A also satisfies the identity $e_9 = 0$.

Proof. To prove the first claim, it is easily checked by hand that:

$$e_3(x,y) = \frac{1}{4}t_3(xy,y,y) - \frac{1}{4}t_3(y^2,x,y) + \frac{1}{4}[t_3(x,y,y)\cdot y] + \frac{1}{12}[t_3(y,y,y)\cdot x]$$

Thus if t_3 is identically zero then so is e_3.

The strategy behind the second proof is similar, except we write the e_9 in terms of the t_4 term. However, instead of being a sum of four terms it is rather a sum of 482 terms and thus not checkable by hand. We give this derivation in Appendix A. The actual verification is done via the computer algebra system Macaulay2. □

Remark 2.7. We conclude by remarking that, by direct inspection of the coefficients appearing in the proofs, all results of this paper are true for fields of characteristic greater than 7 and our counterexamples are true for characteristic greater than 2.

References

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Appendices

A. Form of e_9 identity

In this appendix we give an expression for e_9 in terms of t_4. This identity was verified by Macaulay2. This is unlikely to be the shortest length such expression.

$$\begin{aligned}
e_9(x,y) = &\tfrac{81}{1000}[y\cdot[t_4(y,y^2,y^2,y(yx))\cdot y]] \\
&+ \tfrac{293}{4500}[y\cdot[y\cdot[t_4(y,y,y,y(y(yx)))\cdot y]]] \\
&+ \tfrac{43}{700}[t_4(y,y,y(yy^2),y(yx))\cdot y] \\
&+ \tfrac{53}{875}[y\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y,yx)\cdot y]]]]] \\
&+ \tfrac{83}{1400}[y\cdot[t_4(y,y,y(y(yy^2)),x)\cdot y]] \\
&- \tfrac{41}{700}[t_4(y,y^2,yy^2,y(yx))\cdot y] \\
&- \tfrac{7}{120}[t_4(y,y,y(y(yy^2)),yx)\cdot y] \\
&+ \tfrac{1529}{31500}[y\cdot[y\cdot[y\cdot[t_4(y,y,y,y(yx))\cdot y]]]] \\
&+ \tfrac{487}{10500}[t_4(y,y,y^2,y(y(y(yx))))\cdot y] \\
&+ \tfrac{1919}{42000}[y\cdot[t_4(y,y,y^2,y^2(yx))\cdot y]] \\
&- \tfrac{9}{200}[y\cdot[t_4(y,y^2,y(yy^2),x)\cdot y]] \\
&- \tfrac{547}{12600}[y\cdot[t_4(y,y,y,y(y^2(yx)))\cdot y]] \\
&- \tfrac{149}{3500}t_4(y,y,y^2,y(y(y(y(yx)))))
\end{aligned}$$

$$
\begin{aligned}
&- \tfrac{89}{2100}\,[y\cdot[t_4(y,y,y,y^2(y(yx)))\cdot y]]\\
&- \tfrac{573}{14000}\,[y\cdot[y\cdot[t_4(y,y,y^2,yx)\cdot[y\cdot y]]]]\\
&- \tfrac{373}{10500}\,[y\cdot[y\cdot[t_4(y,y,yy^2,yx)\cdot y]]]\\
&+ \tfrac{341}{10500}\,[t_4(y,y,y^2,y(y^2(yx)))\cdot y]\\
&+ \tfrac{409}{12600}\,[y\cdot[y\cdot[[t_4(y,y,y,yx)\cdot y]\cdot[y\cdot y]]]]\\
&+ \tfrac{2033}{63000}\,[y\cdot[t_4(y,y,y,y(yx))\cdot[y\cdot[y\cdot y]]]]\\
&+ \tfrac{29}{900}\,[y\cdot[t_4(y,y,y,y(yy^2))\cdot[y\cdot x]]]\\
&- \tfrac{9}{280}\,t_4(y,y,y(y(y(yy^2))),yx)\\
&- \tfrac{223}{7000}\,[y\cdot[y\cdot[t_4(y,y^2,y^2,yx)\cdot y]]]\\
&+ \tfrac{31}{1050}\,t_4(y,y,y(yy^2),y(y(yx)))\\
&+ \tfrac{407}{14000}\,[y\cdot[y\cdot[y\cdot[t_4(y,y,y^2,x)\cdot[y\cdot y]]]]]\\
&- \tfrac{43}{1500}\,[y\cdot[y\cdot[y\cdot[[t_4(y,y,y,x)\cdot y]\cdot[y\cdot y]]]]]\\
&+ \tfrac{39}{1400}\,[y\cdot[y\cdot[t_4(y,y^2,yy^2,x)\cdot y]]]\\
&- \tfrac{113}{4200}\,[y\cdot[y\cdot[y\cdot[t_4(y,y,y^2,yx)\cdot y]]]]\\
&- \tfrac{2}{75}\,[y\cdot[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot[y\cdot x]]]]]]\\
&+ \tfrac{19}{750}\,[y\cdot[t_4(y,y,y,(y(yy^2))x)\cdot y]]\\
&- \tfrac{113}{4500}\,[t_4(y,y,y,y(y(y(y(yx)))))\cdot y]\\
&+ \tfrac{1}{40}\,t_4(y,y^2,y(y(y(yy^2))),x)\\
&+ \tfrac{87}{3500}\,[y\cdot[y\cdot[y\cdot[t_4(y,y^2,y^2,x)\cdot y]]]]\\
&- \tfrac{62}{2625}\,[y\cdot[t_4(y,y,y,y(y(y(yx))))\cdot y]]\\
&- \tfrac{33}{1400}\,[y\cdot[y\cdot[t_4(y,y,y^2,y(yx))\cdot y]]]\\
&- \tfrac{7}{300}\,t_4(y,y^2,yy^2,y(y(yx)))\\
&- \tfrac{97}{4200}\,[y\cdot[t_4(y,y^2,yy^2,yx)\cdot y]]\\
&- \tfrac{479}{21000}\,[y\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y,x)\cdot[y\cdot y]]]]]]\\
&+ \tfrac{67}{3000}\,[y\cdot[t_4(y,y,yy^2,y(yx))\cdot y]]\\
&+ \tfrac{701}{31500}\,[y\cdot[y\cdot[t_4(y,y,y,y^2(yx))\cdot y]]]\\
&+ \tfrac{1}{45}\,t_4(y,y,y,y(y(y(y(y(yx))))))\\
&- \tfrac{1}{48}\,[t_4(y,y,y^2,y(y(yy^2)))\cdot x]\\
&- \tfrac{257}{12600}\,[y\cdot[y\cdot[t_4(y,y,y,y(yy^2))\cdot x]]]\\
&- \tfrac{271}{14000}\,[t_4(y,y^2,y^2,y^2(yx))\cdot y]\\
&- \tfrac{27}{1400}\,[y\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y^2,x)\cdot y]]]]]\\
&+ \tfrac{2}{105}\,t_4(y,y,y(yy^2),y^2(yx))\\
&+ \tfrac{2}{105}\,t_4(y,y,y(y(yy^2)),y(yx))\\
&+ \tfrac{199}{10500}\,[y\cdot[y\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y,x)\cdot y]]]]]]\\
&- \tfrac{113}{6000}\,[y\cdot[[y\cdot y]\cdot[t_4(y,y,y,yx)\cdot[y\cdot y]]]]\\
&+ \tfrac{169}{9000}\,[y\cdot[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot[y\cdot x]]]]]\\
&- \tfrac{7}{375}\,[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot[y\cdot[y\cdot x]]]]]]\\
&- \tfrac{13}{700}\,[y\cdot[y\cdot[y\cdot[t_4(y,y,y,x)\cdot[y\cdot[y\cdot y]]]]]]
\end{aligned}
$$

$$
\begin{aligned}
&+ \tfrac{97}{5250}\, t_4(y, y^2, y^2, y(y(y(yx))))\\
&- \tfrac{1}{56}\, [t_4(y, y, y(y(y(yy^2))), x)\cdot y]\\
&- \tfrac{77}{4500}\, [y\cdot[y\cdot[t_4(y, y, y, y^2)\cdot[y\cdot[y\cdot x]]]]]\\
&+ \tfrac{1}{60}\, t_4(y, y, y(y(y(yy^2)))), x)\\
&+ \tfrac{33}{2000}\, [y\cdot[t_4(y, y^2, y^2, yx)\cdot[y\cdot y]]]\\
&+ \tfrac{229}{14000}\, [y\cdot[t_4(y, y, yy^2, yx)\cdot[y\cdot y]]]\\
&- \tfrac{997}{63000}\, [y\cdot[[t_4(y, y, y, y)\cdot y]\cdot[y\cdot[y\cdot[y\cdot x]]]]]\\
&+ \tfrac{124}{7875}\, [y\cdot[y\cdot[y\cdot[t_4(y, y, y, y^2)\cdot[y\cdot x]]]]]\\
&+ \tfrac{31}{2000}\, [y\cdot[y\cdot[t_4(y, y, y^2, x)\cdot[y\cdot[y\cdot y]]]]]\\
&+ \tfrac{27}{1750}\, [y\cdot[t_4(y, y, y, yy^2)\cdot[y\cdot[y\cdot x]]]]\\
&+ \tfrac{3}{200}\, [y\cdot[y\cdot[t_4(y, y, y, x)\cdot[y\cdot[y\cdot[y\cdot y]]]]]]\\
&+ \tfrac{3}{200}\, [y\cdot[t_4(y, y, y(yy^2), x)\cdot[y\cdot y]]]\\
&- \tfrac{227}{15750}\, t_4(y, y, y, y^2(y(y(y(yx)))))\\
&+ \tfrac{47}{3500}\, [y\cdot[y\cdot[t_4(y, y, y, y(y^2x))\cdot y]]]\\
&- \tfrac{137}{10500}\, [t_4(y, y, yy^2, y(y(yx)))\cdot y]\\
&+ \tfrac{179}{14000}\, [y\cdot[t_4(y, y, y^2, y(yx))\cdot[y\cdot y]]]\\
&+ \tfrac{1}{80}\, [[y\cdot[t_4(y, y, y^2, yy^2)\cdot y]]\cdot x]\\
&+ \tfrac{521}{42000}\, [y\cdot[y\cdot[[y\cdot y]\cdot[t_4(y, y, y, y)\cdot[y\cdot x]]]]]\\
&- \tfrac{43}{3500}\, t_4(y, y, yy^2, y(y(y(yx))))\\
&+ \tfrac{109}{9000}\, [y\cdot[y\cdot[y\cdot[t_4(y, y, y, yx)\cdot[y\cdot y]]]]]\\
&- \tfrac{33}{2800}\, [y\cdot[y\cdot[t_4(y, y, y^2, y^2x)\cdot y]]]\\
&- \tfrac{367}{31500}\, [t_4(y, y, y, y^2(y(y(yx))))\cdot y]\\
&- \tfrac{479}{42000}\, [y\cdot[y\cdot[y\cdot[y\cdot[t_4(y, y, y, y^2)\cdot x]]]]]\\
&+ \tfrac{89}{7875}\, [y\cdot[y\cdot[y\cdot[y\cdot[t_4(y, y, y, y)\cdot[y\cdot x]]]]]]\\
&- \tfrac{47}{4200}\, t_4(y, y, y, (y(y(y(yy^2))))x)\\
&+ \tfrac{39}{3500}\, [y\cdot[t_4(y, y, y^2, y(y^2x))\cdot y]]\\
&+ \tfrac{173}{15750}\, [y\cdot[t_4(y, y, y, y(y(yx)))\cdot[y\cdot y]]]\\
&- \tfrac{229}{21000}\, [y\cdot[[y\cdot y]\cdot[y\cdot[t_4(y, y, y, yx)\cdot y]]]]\\
&- \tfrac{113}{10500}\, [y\cdot[y\cdot[y\cdot[t_4(y, y, y, y^2x)\cdot y]]]]\\
&- \tfrac{19}{1800}\, [[y\cdot[y\cdot[t_4(y, y, y, yy^2)\cdot y]]]\cdot x]\\
&- \tfrac{31}{3000}\, [y\cdot[y\cdot[[t_4(y, y, y, x)\cdot y]\cdot[y\cdot[y\cdot y]]]]]\\
&+ \tfrac{43}{4200}\, [y\cdot[y\cdot[t_4(y, y, y, yy^2)\cdot[y\cdot x]]]]\\
&+ \tfrac{23}{2250}\, [t_4(y, y, y, y(y(yx)))\cdot[y\cdot[y\cdot y]]]\\
&- \tfrac{1}{100}\, [t_4(y, y, y(yy^2), y^2x)\cdot y]\\
&+ \tfrac{157}{15750}\, [[t_4(y, y, y, y(y(yx)))\cdot y]\cdot[y\cdot y]]\\
&- \tfrac{89}{9000}\, [y\cdot[y\cdot[[t_4(y, y, y, y)\cdot y]\cdot[y\cdot[y\cdot x]]]]]\\
&- \tfrac{41}{4200}\, [y\cdot[[y\cdot y]\cdot[t_4(y, y, y, y)\cdot[y\cdot[y\cdot x]]]]]\\
&- \tfrac{407}{42000}\, [y\cdot[y\cdot[y\cdot[[y\cdot y]\cdot[t_4(y, y, y, y)\cdot x]]]]]
\end{aligned}
$$

$$
\begin{aligned}
&+ \tfrac{101}{10500}\, t_4(y,y,y,y(y(y^2(y(yx)))))\\
&- \tfrac{149}{15750}\, [y\cdot[y\cdot[t_4(y,y,y,y(yx))\cdot[y\cdot y]]]]\\
&+ \tfrac{59}{6300}\, [y\cdot[y\cdot[[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]\cdot x]]]\\
&+ \tfrac{59}{6300}\, [y\cdot[y\cdot[[t_4(y,y,y,y^2)\cdot[y\cdot y]]\cdot x]]]\\
&- \tfrac{29}{3150}\, [y\cdot[[t_4(y,y,y,yx)\cdot y]\cdot[y\cdot[y\cdot y]]]]\\
&- \tfrac{8}{875}\, [y\cdot[y\cdot[y\cdot[t_4(y,y,yy^2,x)\cdot y]]]]\\
&- \tfrac{191}{21000}\, t_4(y,y,y^2,y(y^2(y(yx))))\\
&- \tfrac{19}{2100}\, t_4(y,y,y,y(y^2(y^2(yx))))\\
&- \tfrac{2}{225}\, [y\cdot[[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]\cdot[y\cdot x]]]\\
&- \tfrac{2}{225}\, [y\cdot[[t_4(y,y,y,y^2)\cdot[y\cdot y]]\cdot[y\cdot x]]]\\
&- \tfrac{31}{3500}\, [t_4(y,y^2,y^2,y(y(yx)))\cdot y]\\
&+ \tfrac{223}{25200}\, [y\cdot[y\cdot[[y\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]\cdot x]]]\\
&- \tfrac{223}{25200}\, [y\cdot[y\cdot[t_4(y,y^2,y^2,y^2)\cdot x]]]\\
&+ \tfrac{37}{4200}\, [y\cdot[t_4(y,y,y^2,yx)\cdot[y\cdot[y\cdot y]]]]\\
&- \tfrac{79}{9000}\, [y\cdot[y\cdot[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot[y\cdot x]]]]]\\
&+ \tfrac{13}{1500}\, t_4(y,y,y,(y(yy^2))(y(yx)))\\
&- \tfrac{3}{350}\, [[y\cdot y]\cdot[t_4(y,y,y^2,yx)\cdot[y\cdot y]]]\\
&- \tfrac{179}{21000}\, [y\cdot[y\cdot[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot x]]]]]]\\
&+ \tfrac{47}{5600}\, t_4(y,y,y^2,y^2(y^2(yx)))\\
&- \tfrac{117}{14000}\, [t_4(y,y,y^2,y^2(y^2x))\cdot y]\\
&- \tfrac{1}{120}\, [t_4(y,y,y,x)\cdot[y\cdot[y\cdot[y\cdot[y\cdot[y\cdot y]]]]]]\\
&+ \tfrac{1}{120}\, [[t_4(y,y,y^2,yy^2)\cdot[y\cdot y]]\cdot x]\\
&+ \tfrac{1}{120}\, [[t_4(y,y,yy^2,yy^2)\cdot y]\cdot x]\\
&- \tfrac{1}{120}\, [t_4(y,y,yy^2,y(yy^2))\cdot x]\\
&- \tfrac{1}{120}\, [t_4(y,y,y(yy^2),yx)\cdot[y\cdot y]]\\
&+ \tfrac{517}{63000}\, [y\cdot[t_4(y,y,y,y^2)\cdot[[y\cdot y]\cdot[y\cdot x]]]]\\
&- \tfrac{257}{31500}\, t_4(y,y,y,y(y^2(y(y(yx)))))\\
&+ \tfrac{17}{2100}\, t_4(y,y^2,y^2,y(y^2(yx)))\\
&- \tfrac{29}{3600}\, [y\cdot[[y\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]\cdot[y\cdot x]]]\\
&+ \tfrac{29}{3600}\, [y\cdot[t_4(y,y^2,y^2,y^2)\cdot[y\cdot x]]]\\
&+ \tfrac{253}{31500}\, [t_4(y,y,y,y)\cdot[y\cdot[y\cdot[y\cdot[y\cdot[y\cdot x]]]]]]\\
&- \tfrac{83}{10500}\, [t_4(y,y,y,y((y(yy^2))x))\cdot y]\\
&+ \tfrac{11}{1400}\, [y\cdot[t_4(y,y,y^2,y(y(yx)))\cdot y]]\\
&- \tfrac{11}{1400}\, [t_4(y,y^2,y^2,y(y^2x))\cdot y]\\
&- \tfrac{107}{14000}\, [y\cdot[t_4(y,y^2,y^2,y^2x)\cdot y]]\\
&- \tfrac{47}{6300}\, [t_4(y,y,y,y(y(y^2(yx))))\cdot y]\\
&- \tfrac{11}{1500}\, [y\cdot[[y\cdot y]\cdot[t_4(y,y,y,y^2)\cdot[y\cdot x]]]]\\
&- \tfrac{61}{8400}\, [t_4(y,y,y,(y(y(yy^2)))x)\cdot y]
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{99}{14000} [y \cdot [t_4(y,y,y,y) \cdot [y \cdot [[y \cdot y] \cdot [y \cdot x]]]]] \\
&- \tfrac{99}{14000} [y \cdot [t_4(y,y^2,y^2,x) \cdot [y \cdot [y \cdot y]]]] \\
&- \tfrac{11}{1575} [t_4(y,y,y,y(yy^2)) \cdot [[y \cdot y] \cdot x]] \\
&+ \tfrac{11}{1575} [[y \cdot y] \cdot [[t_4(y,y,y,yx) \cdot y] \cdot [y \cdot y]]] \\
&+ \tfrac{1}{144} [[t_4(y,y,y,y) \cdot [y \cdot [y \cdot [y \cdot [y \cdot y]]]]] \cdot x] \\
&+ \tfrac{29}{4200} [y \cdot [t_4(y,y,y(yy^2),yx) \cdot y]] \\
&- \tfrac{71}{10500} t_4(y,y,yy^2,y^2(y(yx))) \\
&- \tfrac{11}{1680} t_4(y,y^2,yy^2,y^2(yx)) \\
&+ \tfrac{17}{2625} [y \cdot [y \cdot [y \cdot [t_4(y,y,y,y) \cdot [[y \cdot y] \cdot x]]]]] \\
&- \tfrac{9}{1400} [y \cdot [y \cdot [t_4(y,y,y(yy^2),x) \cdot y]]] \\
&- \tfrac{11}{1750} [y \cdot [t_4(y,y,yy^2,y^2x) \cdot y]] \\
&- \tfrac{79}{12600} t_4(y,y,y,y^2(y(y^2(yx)))) \\
&+ \tfrac{1}{160} [y \cdot [[y \cdot y] \cdot [t_4(y,y,y^2,x) \cdot [y \cdot y]]]] \\
&- \tfrac{43}{7000} [y \cdot [[t_4(y,y,y,y) \cdot [y \cdot y]] \cdot [y \cdot [y \cdot x]]]] \\
&+ \tfrac{43}{7000} [y \cdot [t_4(y,y,y^2,y^2) \cdot [y \cdot [y \cdot x]]]] \\
&- \tfrac{187}{31500} [[t_4(y,y,y,y) \cdot y] \cdot [y \cdot [y \cdot [y \cdot [y \cdot x]]]]] \\
&+ \tfrac{37}{6300} t_4(y,y,y,(y(y(yy^2)))(yx)) \\
&- \tfrac{7}{1200} t_4(y,y,y^2,y(y(y^2(yx)))) \\
&+ \tfrac{61}{10500} [t_4(y,y,y,yy^2) \cdot [y \cdot [y \cdot [y \cdot x]]]] \\
&- \tfrac{73}{12600} [[y \cdot y] \cdot [t_4(y,y,y,y(yy^2)) \cdot x]] \\
&- \tfrac{121}{21000} [y \cdot [y \cdot [y \cdot [t_4(y,y,y,yy^2) \cdot x]]]] \\
&- \tfrac{1}{180} [[[t_4(y,y,y,yy^2) \cdot y] \cdot [y \cdot y]] \cdot x] \\
&- \tfrac{1}{180} [[y \cdot [t_4(y,y,y,y(yy^2)) \cdot y]] \cdot x] \\
&+ \tfrac{1}{180} [t_4(y,y,y,y(y(y(yy^2)))) \cdot x] \\
&+ \tfrac{1}{180} t_4(y,y,y,y((y(yy^2))(yx))) \\
&+ \tfrac{233}{42000} [t_4(y,y,y,y(y^2(y^2x))) \cdot y] \\
&- \tfrac{11}{2000} [y \cdot [[y \cdot y] \cdot [y \cdot [t_4(y,y,y,y) \cdot [y \cdot x]]]]] \\
&+ \tfrac{17}{3150} [t_4(y,y,y,y^2(y^2(yx))) \cdot y] \\
&- \tfrac{3}{560} [y \cdot [t_4(y,y^2,yy^2,x) \cdot [y \cdot y]]] \\
&+ \tfrac{37}{7000} [y \cdot [y \cdot [y \cdot [[t_4(y,y,y,y) \cdot [y \cdot y]] \cdot x]]]] \\
&- \tfrac{37}{7000} [y \cdot [y \cdot [y \cdot [t_4(y,y,y^2,y^2) \cdot x]]]] \\
&- \tfrac{31}{6000} [y \cdot [y \cdot [[y \cdot [y \cdot y]] \cdot [t_4(y,y,y,y) \cdot x]]]] \\
&- \tfrac{323}{63000} [y \cdot [y \cdot [y \cdot [[t_4(y,y,y,y) \cdot y] \cdot [y \cdot x]]]]] \\
&- \tfrac{43}{8400} t_4(y,y,yy^2,y(y^2(yx))) \\
&+ \tfrac{319}{63000} [y \cdot [y \cdot [t_4(y,y,y,yx) \cdot [y \cdot [y \cdot y]]]]] \\
&+ \tfrac{1}{200} [[y \cdot [y \cdot [y \cdot [t_4(y,y,y,y) \cdot [y \cdot y]]]]] \cdot x] \\
&- \tfrac{1}{200} [[y \cdot [y \cdot [t_4(y,y,y^2,y^2) \cdot y]]] \cdot x] \\
&- \tfrac{1}{200} [t_4(y,y^2,y(yy^2),x) \cdot [y \cdot y]]
\end{aligned}
$$

$$- \tfrac{13}{2625}\, [y \cdot [t_4(y, y, y, y(y(y^2 x))) \cdot y]]$$
$$- \tfrac{103}{21000}\, [t_4(y, y, y^2, y(y(yx))) \cdot [y \cdot y]]$$
$$- \tfrac{281}{63000}\, [t_4(y, y, y, (y(yy^2))(yx)) \cdot y]$$
$$+ \tfrac{1}{225}\, [[y \cdot [y \cdot [y \cdot [t_4(y, y, y, y^2) \cdot y]]]] \cdot x]$$
$$+ \tfrac{137}{31500}\, [[y \cdot y] \cdot [t_4(y, y, y, y(yx)) \cdot [y \cdot y]]]$$
$$- \tfrac{3}{700}\, t_4(y, y, y(yy^2), y(y^2 x))$$
$$+ \tfrac{179}{42000}\, [y \cdot [[t_4(y, y, y, x) \cdot [y \cdot y]] \cdot [y \cdot [y \cdot y]]]]$$
$$+ \tfrac{67}{15750}\, t_4(y, y, y, y(y(y(y^2(yx)))))$$
$$- \tfrac{1}{240}\, [[y \cdot [y \cdot [t_4(y, y, y, y) \cdot [y \cdot [y \cdot y]]]]] \cdot x]$$
$$+ \tfrac{1}{240}\, [[y \cdot [y \cdot [[t_4(y, y, y, y) \cdot y] \cdot [y \cdot y]]]] \cdot x]$$
$$- \tfrac{1}{240}\, [[y \cdot [y \cdot [t_4(y, y, y, y^2) \cdot [y \cdot y]]]] \cdot x]$$
$$- \tfrac{29}{7000}\, t_4(y, y, y^2, y(y(y(y^2 x))))$$
$$- \tfrac{43}{10500}\, [t_4(y, y, y^2, y^2(y(yx))) \cdot y]$$
$$- \tfrac{167}{42000}\, [t_4(y, y, yy^2, y^2(yx)) \cdot y]$$
$$+ \tfrac{2}{525}\, [y \cdot [y \cdot [[t_4(y, y, y, y) \cdot [y \cdot y]] \cdot [y \cdot x]]]]$$
$$- \tfrac{2}{525}\, [y \cdot [y \cdot [t_4(y, y, y^2, y^2) \cdot [y \cdot x]]]]$$
$$+ \tfrac{2}{525}\, t_4(y, y, y^2, y^2(y(y(yx))))$$
$$+ \tfrac{2}{525}\, [t_4(y, y^2, y^2, yx) \cdot [y \cdot [y \cdot y]]]$$
$$- \tfrac{2}{525}\, [t_4(y, y^2, yy^2, yx) \cdot [y \cdot y]]$$
$$+ \tfrac{53}{14000}\, [y \cdot [y \cdot [t_4(y, y, yy^2, x) \cdot [y \cdot y]]]]$$
$$- \tfrac{157}{42000}\, [y \cdot [y \cdot [[y \cdot y] \cdot [t_4(y, y, y, x) \cdot [y \cdot y]]]]]$$
$$- \tfrac{13}{3500}\, [t_4(y, y, y, y(y(y(y^2 x)))) \cdot y]$$
$$+ \tfrac{13}{3500}\, [y \cdot [y \cdot [t_4(y, y^2, y^2, x) \cdot [y \cdot y]]]]$$
$$+ \tfrac{51}{14000}\, [y \cdot [[t_4(y, y, y^2, x) \cdot y] \cdot [y \cdot [y \cdot y]]]]$$
$$- \tfrac{19}{5250}\, [[t_4(y, y, y, yx) \cdot y] \cdot [y \cdot [y \cdot [y \cdot y]]]]$$
$$+ \tfrac{1}{280}\, [y \cdot [[y \cdot y] \cdot [t_4(y, y, y, x) \cdot [y \cdot [y \cdot y]]]]]$$
$$+ \tfrac{1}{280}\, t_4(y, y, y(y(yy^2)), y^2 x)$$
$$- \tfrac{1}{280}\, [t_4(y, y^2, yy^2, x) \cdot [y \cdot [y \cdot y]]]$$
$$- \tfrac{11}{3150}\, [t_4(y, y, y, y(yy^2)) \cdot [y \cdot [y \cdot x]]]$$
$$+ \tfrac{73}{21000}\, [t_4(y, y, y^2, y(yx)) \cdot [y \cdot [y \cdot y]]]$$
$$- \tfrac{29}{8400}\, [t_4(y, y, y^2, yy^2) \cdot [y \cdot [y \cdot x]]]$$
$$- \tfrac{1}{300}\, [y \cdot [[y \cdot [t_4(y, y, y, x) \cdot y]] \cdot [y \cdot [y \cdot y]]]]$$
$$+ \tfrac{1}{300}\, [[t_4(y, y, y, y^2(yx)) \cdot y] \cdot [y \cdot y]]$$
$$- \tfrac{1}{300}\, [t_4(y, y, y, y(y(y(yx)))) \cdot [y \cdot y]]$$
$$+ \tfrac{17}{5250}\, [t_4(y, y, y, y) \cdot [y \cdot [[y \cdot y] \cdot [y \cdot [y \cdot x]]]]]$$
$$+ \tfrac{17}{5250}\, [y \cdot [y \cdot [t_4(y, y, y, y^2) \cdot [[y \cdot y] \cdot x]]]]$$
$$+ \tfrac{11}{3500}\, [t_4(y, y^2, y^2, y(yx)) \cdot [y \cdot y]]$$
$$+ \tfrac{131}{42000}\, [y \cdot [y \cdot [y \cdot [y \cdot [[t_4(y, y, y, y) \cdot y] \cdot x]]]]]$$

$$
\begin{aligned}
&- \tfrac{31}{10500}\,[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot[y\cdot[y\cdot[y\cdot x]]]]\\
&+ \tfrac{31}{10500}\,t_4(y,y,y,y(y(y(y(y^2x)))))\\
&+ \tfrac{31}{10500}\,[t_4(y,y,y^2,y^2)\cdot[y\cdot[y\cdot[y\cdot x]]]]\\
&- \tfrac{73}{25200}\,[t_4(y,y,y,y(y^2(yx)))\cdot[y\cdot y]]\\
&+ \tfrac{1}{350}\,[[y\cdot y]\cdot[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[y\cdot x]]]]\\
&+ \tfrac{17}{6000}\,[y\cdot[t_4(y,y,y,y^2x)\cdot[y\cdot[y\cdot y]]]]\\
&- \tfrac{47}{16800}\,[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot[[y\cdot y]\cdot[y\cdot x]]]]\\
&+ \tfrac{1}{360}\,[[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[y\cdot y]]]]\cdot x]\\
&- \tfrac{1}{360}\,[[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]]\cdot x]\\
&+ \tfrac{1}{360}\,[[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]]\cdot x]\\
&- \tfrac{1}{360}\,[[y\cdot[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot y]]]]\cdot x]\\
&- \tfrac{1}{360}\,[[[y\cdot y]\cdot[t_4(y,y,y,y^2)\cdot[y\cdot y]]]\cdot x]\\
&+ \tfrac{1}{360}\,[[t_4(y,y,y,y(yy^2))\cdot[y\cdot y]]\cdot x]\\
&+ \tfrac{1}{360}\,[[t_4(y,y,y,y(y(yy^2)))\cdot y]\cdot x]\\
&- \tfrac{1}{360}\,[t_4(y,y,y,y^2(y(yy^2)))\cdot x]\\
&+ \tfrac{23}{8400}\,[y\cdot[y\cdot[t_4(y,y,y,y^2x)\cdot[y\cdot y]]]]\\
&+ \tfrac{17}{6300}\,[y\cdot[[t_4(y,y,y,y(yy^2))\cdot y]\cdot x]]\\
&- \tfrac{11}{4200}\,[t_4(y,y,yy^2,yx)\cdot[y\cdot[y\cdot y]]]\\
&+ \tfrac{4}{1575}\,[[y\cdot y]\cdot[t_4(y,y,y,yx)\cdot[y\cdot[y\cdot y]]]]\\
&- \tfrac{53}{21000}\,[y\cdot[y\cdot[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,x)\cdot y]]]]]\\
&+ \tfrac{1}{400}\,[y\cdot[t_4(y,y,y^2,yy^2)\cdot[y\cdot x]]]\\
&+ \tfrac{41}{16800}\,[[y\cdot y]\cdot[t_4(y,y,y^2,yy^2)\cdot x]]\\
&- \tfrac{17}{7000}\,[[y\cdot[y\cdot y]]\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot x]]]]\\
&+ \tfrac{17}{7000}\,[[t_4(y,y,y^2,x)\cdot y]\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&+ \tfrac{17}{7000}\,[t_4(y,y,y^2,yx)\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&+ \tfrac{17}{7000}\,t_4(y,y^2,y^2,y^2(y^2x))\\
&- \tfrac{1}{420}\,[y\cdot[[y\cdot y]\cdot[[t_4(y,y,y,x)\cdot y]\cdot[y\cdot y]]]]\\
&+ \tfrac{1}{420}\,[[y\cdot[y\cdot y]]\cdot[t_4(y,y,y,x)\cdot[y\cdot[y\cdot y]]]]\\
&+ \tfrac{29}{12600}\,[[t_4(y,y,y,yy^2)\cdot y]\cdot[y\cdot[y\cdot x]]]\\
&- \tfrac{2}{875}\,t_4(y,y,y^2,y(y^2(y^2x)))\\
&- \tfrac{19}{8400}\,[[t_4(y,y,y^2,yy^2)\cdot y]\cdot[y\cdot x]]\\
&- \tfrac{71}{31500}\,[[t_4(y,y,y,y(yx))\cdot y]\cdot[y\cdot[y\cdot y]]]\\
&+ \tfrac{17}{7875}\,[[y\cdot y]\cdot[t_4(y,y,y,yy^2)\cdot[y\cdot x]]]\\
&+ \tfrac{3}{1400}\,[t_4(y,y,y,y^2(y(y^2x)))\cdot y]\\
&- \tfrac{3}{1400}\,t_4(y,y,y,y((y(y(yy^2)))x))\\
&+ \tfrac{3}{1400}\,[t_4(y,y^2,yy^2,y^2x)\cdot y]\\
&+ \tfrac{179}{84000}\,[y\cdot[[y\cdot[y\cdot y]]\cdot[t_4(y,y,y,y^2)\cdot x]]]\\
&- \tfrac{11}{5250}\,t_4(y,y,y,y(y((y(yy^2))x)))
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{1}{480}\,[y\cdot[[y\cdot y]\cdot[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot x]]]]\\
&- \tfrac{43}{21000}\,[y\cdot[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[y\cdot[y\cdot x]]]]\\
&- \tfrac{43}{21000}\,[y\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot[y\cdot x]]]]\\
&+ \tfrac{17}{8400}\,[[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]\cdot[y\cdot[y\cdot x]]]\\
&+ \tfrac{17}{8400}\,[[t_4(y,y,y,y^2)\cdot[y\cdot y]]\cdot[y\cdot[y\cdot x]]]\\
&- \tfrac{1}{500}\,[[y\cdot[t_4(y,y,y,x)\cdot y]]\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&- \tfrac{1}{500}\,[t_4(y,y,y,y(yx))\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&+ \tfrac{83}{42000}\,[y\cdot[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot[[y\cdot y]\cdot x]]]]\\
&+ \tfrac{41}{21000}\,[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot[[y\cdot y]\cdot x]]]]]\\
&- \tfrac{7}{3600}\,[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[[y\cdot y]\cdot[y\cdot x]]]]\\
&+ \tfrac{7}{3600}\,[[y\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot y]]]\cdot x]\\
&- \tfrac{1}{525}\,[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot[[y\cdot y]\cdot[y\cdot x]]]\\
&+ \tfrac{1}{525}\,[y\cdot[t_4(y,y,y,y^2)\cdot[y\cdot[[y\cdot y]\cdot x]]]]\\
&+ \tfrac{1}{525}\,[t_4(y,y,y,y(y(yy^2)))\cdot[y\cdot x]]\\
&- \tfrac{1}{525}\,[y\cdot[t_4(y,y,y,y(y(yy^2)))\cdot x]]\\
&+ \tfrac{1}{525}\,[t_4(y,y,y^2,y^2)\cdot[[y\cdot y]\cdot[y\cdot x]]]\\
&- \tfrac{157}{84000}\,[y\cdot[y\cdot[[y\cdot y]\cdot[t_4(y,y,y,y^2)\cdot x]]]]\\
&+ \tfrac{47}{25200}\,[[y\cdot y]\cdot[[y\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]\cdot x]]\\
&- \tfrac{47}{25200}\,[[y\cdot y]\cdot[t_4(y,y^2,y^2,y^2)\cdot x]]\\
&- \tfrac{13}{7000}\,[y\cdot[y\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y,y)\cdot x]]]]]]\\
&+ \tfrac{13}{7000}\,[[y\cdot y]\cdot[y\cdot[y\cdot[t_4(y,y,y^2,x)\cdot y]]]]\\
&+ \tfrac{11}{6000}\,[y\cdot[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot x]]]]\\
&- \tfrac{23}{12600}\,[y\cdot[t_4(y,y,y,y^2(yx))\cdot[y\cdot y]]]\\
&+ \tfrac{1}{560}\,[y\cdot[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,x)\cdot[y\cdot y]]]]]\\
&+ \tfrac{1}{560}\,t_4(y,y,y^2,(y(y(yy^2)))x)\\
&+ \tfrac{37}{21000}\,[y\cdot[y\cdot[y\cdot[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot x]]]]\\
&+ \tfrac{37}{21000}\,[y\cdot[y\cdot[y\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot x]]]]\\
&+ \tfrac{37}{21000}\,[[t_4(y,y,y,y^2x)\cdot y]\cdot[y\cdot[y\cdot y]]]\\
&- \tfrac{221}{126000}\,[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot[[y\cdot y]\cdot[y\cdot x]]]]\\
&+ \tfrac{11}{6300}\,[[y\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]\cdot[[y\cdot y]\cdot x]]\\
&+ \tfrac{11}{6300}\,[[t_4(y,y,y,y(yy^2))\cdot y]\cdot[y\cdot x]]\\
&- \tfrac{11}{6300}\,[t_4(y,y^2,y^2,y^2)\cdot[[y\cdot y]\cdot x]]\\
&+ \tfrac{29}{16800}\,[t_4(y,y,y^2,yy^2)\cdot[[y\cdot y]\cdot x]]\\
&- \tfrac{3}{1750}\,t_4(y,y,y,y^2(y^2(y(yx))))\\
&+ \tfrac{107}{63000}\,[[y\cdot[t_4(y,y,y,yy^2)\cdot y]]\cdot[y\cdot x]]\\
&+ \tfrac{1}{600}\,[[y\cdot[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]]\cdot x]\\
&+ \tfrac{1}{600}\,[[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot[y\cdot[y\cdot y]]]\cdot x]\\
&- \tfrac{1}{600}\,[y\cdot[[t_4(y,y,y,yy^2)\cdot y]\cdot[y\cdot x]]]
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{1}{600}\,[[y\cdot[t_4(y,y,y,yy^2)\cdot[y\cdot y]]]\cdot x]\\
&+ \tfrac{1}{600}\,[[y\cdot y]\cdot[t_4(y,y,y,x)\cdot[y\cdot[y\cdot[y\cdot y]]]]]\\
&+ \tfrac{1}{600}\,[t_4(y,y,y,(y(yy^2))x)\cdot[y\cdot y]]\\
&- \tfrac{1}{600}\,[[t_4(y,y,y^2,y^2)\cdot[y\cdot[y\cdot y]]]\cdot x]\\
&- \tfrac{1}{600}\,[[y\cdot[t_4(y,y,y^2,y^2)\cdot[y\cdot y]]]\cdot x]\\
&- \tfrac{41}{25200}\,[[y\cdot y]\cdot[[t_4(y,y,y,yy^2)\cdot y]\cdot x]]\\
&+ \tfrac{17}{10500}\,[t_4(y,y,yy^2,y(yx))\cdot[y\cdot y]]\\
&+ \tfrac{11}{7000}\,[y\cdot[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot[y\cdot[y\cdot x]]]]]\\
&+ \tfrac{11}{7000}\,[t_4(y,y,y^2,y(y(y^2x)))\cdot y]\\
&- \tfrac{13}{8400}\,[y\cdot[y\cdot[t_4(y,y,y^2,yy^2)\cdot x]]]\\
&- \tfrac{4}{2625}\,[t_4(y,y,y,y(y^2x))\cdot[y\cdot[y\cdot y]]]\\
&+ \tfrac{19}{12600}\,[t_4(y,y,y,y^2(yx))\cdot[y\cdot[y\cdot y]]]\\
&- \tfrac{3}{2000}\,[t_4(y,y^2,y^2,y^2x)\cdot[y\cdot y]]\\
&- \tfrac{47}{31500}\,[[y\cdot[y\cdot y]]\cdot[y\cdot[t_4(y,y,y,y)\cdot[y\cdot x]]]]\\
&+ \tfrac{23}{15750}\,[t_4(y,y,y,y(y^2(y(yx)))))\cdot y]\\
&+ \tfrac{61}{42000}\,[y\cdot[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[[y\cdot y]\cdot x]]]]]\\
&- \tfrac{1}{700}\,[y\cdot[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot x]]]\\
&+ \tfrac{1}{700}\,[y\cdot[t_4(y,y,y,y^2(y^2x))\cdot y]]\\
&+ \tfrac{1}{700}\,[y\cdot[[y\cdot y]\cdot[t_4(y,y,y^2,y^2)\cdot x]]]\\
&+ \tfrac{1}{700}\,[[y\cdot y]\cdot[y\cdot[t_4(y,y,y^2,x)\cdot[y\cdot y]]]]\\
&+ \tfrac{1}{700}\,[t_4(y,y,y(y(yy^2)),x)\cdot[y\cdot y]]\\
&+ \tfrac{1}{700}\,t_4(y,y^2,yy^2,y(y^2x))\\
&+ \tfrac{39}{28000}\,[y\cdot[t_4(y,y,y^2,y^2x)\cdot[y\cdot y]]]\\
&- \tfrac{1}{720}\,[[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[y\cdot[y\cdot y]]]]\cdot x]\\
&+ \tfrac{1}{720}\,[[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot[y\cdot y]]]]\cdot x]\\
&+ \tfrac{19}{14000}\,[y\cdot[t_4(y,y,yy^2,x)\cdot[y\cdot[y\cdot y]]]]\\
&+ \tfrac{1}{750}\,[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot[y\cdot x]]]\\
&+ \tfrac{1}{750}\,[t_4(y,y,y,y^2)\cdot[[y\cdot y]\cdot[y\cdot[y\cdot x]]]]\\
&+ \tfrac{1}{750}\,[[y\cdot y]\cdot[t_4(y,y,y,y^2x)\cdot[y\cdot y]]]\\
&- \tfrac{1}{750}\,[[y\cdot y]\cdot[t_4(y,y,y^2,y^2)\cdot[y\cdot x]]]\\
&+ \tfrac{9}{7000}\,[[t_4(y,y,yy^2,x)\cdot y]\cdot[y\cdot[y\cdot y]]]\\
&- \tfrac{2}{1575}\,[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot[y\cdot[y\cdot[y\cdot x]]]]]\\
&+ \tfrac{2}{1575}\,[y\cdot[y\cdot[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[y\cdot x]]]]\\
&+ \tfrac{2}{1575}\,[y\cdot[y\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot x]]]]\\
&+ \tfrac{13}{10500}\,[t_4(y,y,y,y)\cdot[y\cdot[y\cdot[y\cdot[[y\cdot y]\cdot x]]]]]\\
&- \tfrac{13}{10500}\,[y\cdot[y\cdot[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y)\cdot x]]]]]\\
&- \tfrac{13}{10500}\,[[y\cdot y]\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y,x)\cdot y]]]]]\\
&- \tfrac{13}{10500}\,[[y\cdot y]\cdot[y\cdot[y\cdot[t_4(y,y,y,yx)\cdot y]]]]
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{13}{10500}\,[t_4(y,y,y,y^2(y(yx)))\cdot[y\cdot y]]\\
&+ \tfrac{13}{10500}\,[[t_4(y,y,y^2,yx)\cdot y]\cdot[y\cdot[y\cdot y]]]\\
&- \tfrac{31}{25200}\,[y\cdot[[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]]\cdot x]]\\
&- \tfrac{31}{25200}\,[y\cdot[[y\cdot[t_4(y,y,y,y^2)\cdot[y\cdot y]]]\cdot x]]\\
&+ \tfrac{1}{840}\,[[y\cdot[y\cdot y]]\cdot[t_4(y,y,y,yy^2)\cdot x]]\\
&+ \tfrac{1}{840}\,[[y\cdot[y\cdot y]]\cdot[[t_4(y,y,y,x)\cdot y]\cdot[y\cdot y]]]\\
&+ \tfrac{59}{50400}\,[[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]\cdot[[y\cdot y]\cdot x]]\\
&+ \tfrac{59}{50400}\,[[t_4(y,y,y,y^2)\cdot[y\cdot y]]\cdot[[y\cdot y]\cdot x]]\\
&- \tfrac{29}{25200}\,[[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]\cdot[y\cdot[y\cdot x]]]\\
&- \tfrac{29}{25200}\,[[t_4(y,y,y,yy^2)\cdot y]\cdot[[y\cdot y]\cdot x]]\\
&+ \tfrac{1}{875}\,[y\cdot[[y\cdot[y\cdot y]]\cdot[y\cdot[t_4(y,y,y,y)\cdot x]]]]\\
&+ \tfrac{1}{875}\,[[y\cdot[y\cdot y]]\cdot[y\cdot[t_4(y,y,y,x)\cdot[y\cdot y]]]]\\
&- \tfrac{1}{875}\,t_4(y,y,y,y^2(y^2(y^2x)))\\
&- \tfrac{71}{63000}\,[[t_4(y,y,y,y)\cdot y]\cdot[[y\cdot y]\cdot[y\cdot[y\cdot x]]]]\\
&- \tfrac{1}{900}\,[[y\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y,y)\cdot y]]]]]\cdot x]\\
&+ \tfrac{1}{900}\,[[t_4(y,y,y,yy^2)\cdot[y\cdot[y\cdot y]]]\cdot x]\\
&+ \tfrac{23}{21000}\,[y\cdot[t_4(y,y,y,y(y^2x))\cdot[y\cdot y]]]\\
&- \tfrac{17}{15750}\,[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y(yx))\cdot y]]]\\
&- \tfrac{3}{2800}\,t_4(y,y,y^2,y^2(y(y^2x)))\\
&+ \tfrac{53}{50400}\,[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]\cdot x]]\\
&+ \tfrac{53}{50400}\,[[y\cdot y]\cdot[[t_4(y,y,y,y^2)\cdot[y\cdot y]]\cdot x]]\\
&- \tfrac{11}{10500}\,[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot x]]]]]\\
&- \tfrac{13}{12600}\,[[y\cdot[y\cdot y]]\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot x]]]\\
&+ \tfrac{13}{12600}\,[y\cdot[y\cdot[[t_4(y,y,y,yy^2)\cdot y]\cdot x]]]\\
&+ \tfrac{13}{12600}\,[y\cdot[[t_4(y,y^2,y^2,y^2)\cdot y]\cdot x]]\\
&+ \tfrac{1}{1000}\,[y\cdot[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot[[y\cdot y]\cdot x]]]\\
&- \tfrac{1}{1000}\,[y\cdot[t_4(y,y,y^2,y^2)\cdot[[y\cdot y]\cdot x]]]\\
&- \tfrac{1}{1000}\,[t_4(y,y,yy^2,y^2x)\cdot[y\cdot y]]\\
&- \tfrac{31}{31500}\,[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[y\cdot[y\cdot[y\cdot x]]]]\\
&- \tfrac{31}{31500}\,[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot[y\cdot[y\cdot x]]]]\\
&- \tfrac{1}{1050}\,[[y\cdot y]\cdot[y\cdot[[t_4(y,y,y,x)\cdot y]\cdot[y\cdot y]]]]\\
&+ \tfrac{1}{1050}\,t_4(y,y,y,y^2((y(yy^2))x))\\
&- \tfrac{13}{14000}\,[t_4(y,y,y^2,y^2x)\cdot[y\cdot[y\cdot y]]]\\
&- \tfrac{13}{14000}\,[t_4(y,y^2,y^2,x)\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&+ \tfrac{23}{25200}\,[t_4(y,y,y,yy^2)\cdot[[y\cdot y]\cdot[y\cdot x]]]\\
&- \tfrac{1}{1120}\,[y\cdot[[y\cdot y]\cdot[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot x]]]]\\
&+ \tfrac{1}{1120}\,[y\cdot[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y^2)\cdot x]]]]\\
&- \tfrac{37}{42000}\,[t_4(y,y,y,y^2)\cdot[[y\cdot y]\cdot[[y\cdot y]\cdot x]]]
\end{aligned}
$$

$$+ \frac{11}{12600} [[y\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]\cdot[y\cdot[y\cdot x]]]$$
$$- \frac{11}{12600} [t_4(y,y^2,y^2,y^2)\cdot[y\cdot[y\cdot x]]]$$
$$- \frac{3}{3500} [[y\cdot[y\cdot y]]\cdot[y\cdot[y\cdot[t_4(y,y,y,x)\cdot y]]]]$$
$$+ \frac{1}{1200} [y\cdot[[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]\cdot[y\cdot x]]]$$
$$- \frac{1}{1200} [[y\cdot[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y)\cdot y]]]]\cdot x]$$
$$+ \frac{1}{1200} [y\cdot[[y\cdot y]\cdot[t_4(y,y,y,yy^2)\cdot x]]]$$
$$+ \frac{41}{50400} [[y\cdot y]\cdot[[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]\cdot x]]$$
$$- \frac{19}{25200} [t_4(y,y,y,y)\cdot[y\cdot[y\cdot[[y\cdot y]\cdot[y\cdot x]]]]]$$
$$- \frac{19}{25200} [[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]]\cdot[y\cdot x]]$$
$$- \frac{19}{25200} [t_4(y,y,y,y^2)\cdot[y\cdot[[y\cdot y]\cdot[y\cdot x]]]]$$
$$+ \frac{1}{1400} [y\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[[y\cdot y]\cdot x]]]]$$
$$- \frac{1}{1400} [t_4(y,y,y^2,y(yy^2))\cdot[y\cdot x]]$$
$$+ \frac{1}{1400} [y\cdot[t_4(y,y,y^2,y(yy^2))\cdot x]]$$
$$- \frac{1}{1400} [t_4(y,y,yy^2,yy^2)\cdot[y\cdot x]]$$
$$+ \frac{1}{1400} [y\cdot[t_4(y,y,yy^2,yy^2)\cdot x]]$$
$$+ \frac{1}{1400} [t_4(y,y,yy^2,y(y^2x))\cdot y]$$
$$+ \frac{1}{1400} [t_4(y,y,y(yy^2),x)\cdot[y\cdot[y\cdot y]]]$$
$$- \frac{29}{42000} [y\cdot[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot[[y\cdot y]\cdot x]]]]$$
$$- \frac{43}{63000} [[y\cdot y]\cdot[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot x]]]]$$
$$+ \frac{17}{25200} [[y\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot y]]]]\cdot[y\cdot x]]$$
$$+ \frac{17}{25200} [[y\cdot[t_4(y,y,y,y^2)\cdot[y\cdot y]]]\cdot[y\cdot x]]$$
$$+ \frac{1}{1500} [[y\cdot y]\cdot[t_4(y,y,y,y^2)\cdot[[y\cdot y]\cdot x]]]$$
$$+ \frac{1}{1500} [[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y^2x)\cdot y]]]$$
$$- \frac{1}{1575} [[y\cdot y]\cdot[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot x]]]]$$
$$- \frac{1}{1575} [[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[[y\cdot y]\cdot[y\cdot x]]]$$
$$- \frac{1}{1575} [[t_4(y,y,y,y^2)\cdot y]\cdot[[y\cdot y]\cdot[y\cdot x]]]$$
$$+ \frac{1}{1575} [[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y^2)\cdot[y\cdot x]]]]$$
$$+ \frac{53}{84000} [y\cdot[y\cdot[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot y]\cdot x]]]]$$
$$- \frac{13}{21000} [[y\cdot y]\cdot[y\cdot[y\cdot[y\cdot[t_4(y,y,y,y)\cdot x]]]]]$$
$$- \frac{13}{21000} t_4(y,y^2,y^2,y^2(y(yx)))$$
$$- \frac{19}{31500} [y\cdot[[y\cdot[y\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]]\cdot x]]$$
$$- \frac{1}{1680} [t_4(y,y,y,y)\cdot[[y\cdot[y\cdot[y\cdot[y\cdot y]]]]\cdot x]]$$
$$+ \frac{1}{1680} [y\cdot[[t_4(y,y,y^2,yy^2)\cdot y]\cdot x]]$$
$$+ \frac{37}{63000} [[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[y\cdot[y\cdot[y\cdot x]]]]]$$
$$- \frac{37}{63000} [y\cdot[[y\cdot[t_4(y,y,y,yy^2)\cdot y]]\cdot x]]$$
$$- \frac{73}{126000} [y\cdot[[y\cdot[y\cdot y]]\cdot[t_4(y,y,y,y)\cdot[y\cdot x]]]]$$
$$+ \frac{29}{50400} [[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]\cdot[[y\cdot y]\cdot x]]$$
$$+ \frac{1}{1750} [[y\cdot y]\cdot[y\cdot[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot x]]]$$

$$
\begin{aligned}
&+\tfrac{1}{1750}\,[[y\cdot[y\cdot y]]\cdot[y\cdot[t_4(y,y,y,y^2)\cdot x]]]\\
&-\tfrac{1}{1750}\,[t_4(y,y,y,y(y(y^2x)))\cdot[y\cdot y]]\\
&-\tfrac{1}{1750}\,[[y\cdot y]\cdot[y\cdot[t_4(y,y,y^2,y^2)\cdot x]]]\\
&+\tfrac{1}{1750}\,[t_4(y,y,y^2,y(y^2x))\cdot[y\cdot y]]\\
&+\tfrac{1}{1750}\,[t_4(y,y,yy^2,x)\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&-\tfrac{1}{1750}\,[[t_4(y,y^2,y^2,x)\cdot y]\cdot[y\cdot[y\cdot y]]]\\
&+\tfrac{1}{1800}\,[[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[y\cdot[y\cdot y]]]\cdot x]\\
&+\tfrac{1}{1800}\,[[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot[y\cdot y]]]\cdot x]\\
&-\tfrac{13}{25200}\,[y\cdot[y\cdot[[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]\cdot x]]]\\
&-\tfrac{4}{7875}\,[[y\cdot[y\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]]\cdot[y\cdot x]]\\
&+\tfrac{1}{2000}\,[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot x]]]]\\
&-\tfrac{1}{2000}\,[[y\cdot[y\cdot[y\cdot y]]]\cdot[t_4(y,y,y,y)\cdot[y\cdot x]]]\\
&-\tfrac{1}{2000}\,[[y\cdot[y\cdot[y\cdot y]]]\cdot[y\cdot[t_4(y,y,y,y)\cdot x]]]\\
&-\tfrac{1}{2100}\,[y\cdot[[y\cdot y]\cdot[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot x]]]\\
&-\tfrac{1}{2100}\,[[y\cdot y]\cdot[y\cdot[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot x]]]]\\
&-\tfrac{1}{2100}\,[y\cdot[[y\cdot y]\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot x]]]\\
&-\tfrac{1}{2100}\,[[t_4(y,y,y,yx)\cdot[y\cdot y]]\cdot[y\cdot[y\cdot y]]]\\
&+\tfrac{1}{2100}\,t_4(y,y,y,y(y(y^2(y^2x))))\\
&-\tfrac{13}{28000}\,[y\cdot[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot x]]]]\\
&+\tfrac{1}{2250}\,[[y\cdot y]\cdot[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[y\cdot x]]]\\
&+\tfrac{1}{2250}\,[[y\cdot y]\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot x]]]\\
&+\tfrac{3}{7000}\,[[t_4(y,y,y,y)\cdot[y\cdot y]]\cdot[y\cdot[[y\cdot y]\cdot x]]]\\
&-\tfrac{3}{7000}\,[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot[[y\cdot y]\cdot x]]]]\\
&+\tfrac{3}{7000}\,[[t_4(y,y,y,x)\cdot[y\cdot y]]\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&+\tfrac{3}{7000}\,[t_4(y,y,y,y^2x)\cdot[y\cdot[y\cdot[y\cdot y]]]]\\
&-\tfrac{3}{7000}\,[t_4(y,y,y,y^2(y^2x))\cdot[y\cdot y]]\\
&-\tfrac{3}{7000}\,[t_4(y,y,y^2,y^2)\cdot[y\cdot[[y\cdot y]\cdot x]]]\\
&-\tfrac{3}{7000}\,[y\cdot[[y\cdot[t_4(y,y,y^2,y^2)\cdot y]]\cdot x]]\\
&+\tfrac{3}{7000}\,[[y\cdot[t_4(y,y,y^2,y^2)\cdot y]]\cdot[y\cdot x]]\\
&+\tfrac{3}{7000}\,t_4(y,y,yy^2,y^2(y^2x))\\
&+\tfrac{3}{7000}\,t_4(y,y^2,y^2,y(y(y^2x)))\\
&-\tfrac{1}{2625}\,[[y\cdot[y\cdot y]]\cdot[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot x]]]\\
&+\tfrac{1}{2625}\,[y\cdot[[y\cdot[y\cdot[t_4(y,y,y,y^2)\cdot y]]]\cdot x]]\\
&-\tfrac{1}{2625}\,[[y\cdot[y\cdot[t_4(y,y,y,y^2)\cdot y]]]\cdot[y\cdot x]]\\
&+\tfrac{1}{2625}\,[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,yy^2)\cdot x]]]\\
&-\tfrac{23}{63000}\,[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[y\cdot x]]]]\\
&+\tfrac{1}{2800}\,[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot[y\cdot[[y\cdot y]\cdot x]]]]\\
&+\tfrac{1}{3000}\,[y\cdot[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[[y\cdot y]\cdot x]]]
\end{aligned}
$$

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\begin{aligned}
&+\tfrac{1}{3000}\,[y\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot[[y\cdot y]\cdot x]]]\\
&+\tfrac{1}{3000}\,t_4(y,y,y,(y(yy^2))(y^2x))\\
&+\tfrac{1}{3150}\,[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,yx)\cdot[y\cdot y]]]]\\
&+\tfrac{13}{42000}\,[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[y\cdot[[y\cdot y]\cdot x]]]]\\
&+\tfrac{13}{42000}\,[[y\cdot[y\cdot y]]\cdot[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot x]]]\\
&+\tfrac{1}{3500}\,[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[y\cdot[[y\cdot y]\cdot x]]]]\\
&-\tfrac{1}{3500}\,[y\cdot[[t_4(y,y,y,yy^2)\cdot[y\cdot y]]\cdot x]]\\
&+\tfrac{1}{3500}\,[[t_4(y,y,y,yy^2)\cdot[y\cdot y]]\cdot[y\cdot x]]\\
&+\tfrac{1}{3500}\,t_4(y,y,yy^2,y(y(y^2x)))\\
&-\tfrac{1}{4200}\,[y\cdot[[t_4(y,y,y,y)\cdot[y\cdot[y\cdot[y\cdot y]]]]\cdot x]]\\
&+\tfrac{1}{4200}\,[y\cdot[[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[y\cdot y]]]\cdot x]]\\
&+\tfrac{1}{4200}\,[[t_4(y,y,y,y)\cdot[y\cdot[y\cdot[y\cdot y]]]]\cdot[y\cdot x]]\\
&-\tfrac{1}{4200}\,[[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot[y\cdot y]]]\cdot[y\cdot x]]\\
&-\tfrac{1}{4200}\,[y\cdot[[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot y]]]\cdot x]]\\
&-\tfrac{1}{4200}\,[[y\cdot[y\cdot y]]\cdot[t_4(y,y,y,y^2)\cdot[y\cdot x]]]\\
&+\tfrac{1}{4200}\,[[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot y]]]\cdot[y\cdot x]]\\
&-\tfrac{1}{4200}\,t_4(y,y,y,(y^2(y(yy^2)))x)\\
&+\tfrac{19}{84000}\,[y\cdot[[y\cdot[y\cdot y]]\cdot[[t_4(y,y,y,y)\cdot y]\cdot x]]]\\
&-\tfrac{1}{4500}\,[t_4(y,y,y,y^2)\cdot[y\cdot[y\cdot[y\cdot[y\cdot x]]]]]\\
&+\tfrac{3}{14000}\,[[y\cdot[y\cdot[y\cdot y]]]\cdot[t_4(y,y,y,y^2)\cdot x]]\\
&+\tfrac{1}{5040}\,[y\cdot[[y\cdot[[t_4(y,y,y,y)\cdot y]\cdot[y\cdot y]]]\cdot x]]\\
&+\tfrac{1}{5250}\,[[y\cdot y]\cdot[y\cdot[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot x]]]\\
&+\tfrac{1}{5250}\,[[y\cdot[y\cdot y]]\cdot[y\cdot[y\cdot[t_4(y,y,y,y)\cdot x]]]]\\
&+\tfrac{1}{5250}\,[[y\cdot y]\cdot[y\cdot[[t_4(y,y,y,y^2)\cdot y]\cdot x]]]\\
&-\tfrac{1}{5250}\,[t_4(y,y,y,yy^2)\cdot[y\cdot[[y\cdot y]\cdot x]]]\\
&-\tfrac{1}{5250}\,[[t_4(y,y,y,y(y^2x))\cdot y]\cdot[y\cdot y]]\\
&+\tfrac{1}{5600}\,[t_4(y,y,y^2,y^2(yx))\cdot[y\cdot y]]\\
&-\tfrac{1}{6000}\,[[y\cdot y]\cdot[[t_4(y,y,y,y)\cdot y]\cdot[[y\cdot y]\cdot x]]]\\
&-\tfrac{1}{7000}\,[y\cdot[[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y)\cdot y]]]\cdot x]]\\
&+\tfrac{1}{7000}\,[[y\cdot[t_4(y,y,y,y)\cdot y]]\cdot[y\cdot[[y\cdot y]\cdot x]]]\\
&+\tfrac{1}{7000}\,[[[y\cdot y]\cdot[y\cdot[t_4(y,y,y,y)\cdot y]]]\cdot[y\cdot x]]\\
&+\tfrac{1}{7000}\,[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot[[y\cdot y]\cdot x]]]\\
&+\tfrac{1}{7000}\,[y\cdot[[t_4(y,y,y,y^2x)\cdot y]\cdot[y\cdot y]]]\\
&-\tfrac{1}{7000}\,t_4(y,y,y,y(y^2(y(y^2x))))\\
&-\tfrac{1}{10500}\,[y\cdot[[y\cdot[y\cdot[y\cdot[t_4(y,y,y,y)\cdot y]]]]\cdot x]]\\
&+\tfrac{1}{10500}\,[[y\cdot[y\cdot[y\cdot y]]]\cdot[[t_4(y,y,y,y)\cdot y]\cdot x]]\\
&+\tfrac{1}{10500}\,[[y\cdot[y\cdot[y\cdot[t_4(y,y,y,y)\cdot y]]]]\cdot[y\cdot x]]\\
&+\tfrac{1}{10500}\,[y\cdot[[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot y]]\cdot x]]
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{1}{10500}\,[[[t_4(y,y,y,y^2)\cdot y]\cdot[y\cdot y]]\cdot[y\cdot x]] \\
&+ \tfrac{1}{10500}\,[y\cdot[[t_4(y,y,y,y(yx))\cdot y]\cdot[y\cdot y]]] \\
&+ \tfrac{1}{10500}\,t_4(y,y,y,y^2(y(y(y^2x)))) \\
&+ \tfrac{1}{12600}\,[[t_4(y,y^2,y^2,y^2)\cdot y]\cdot[y\cdot x]] \\
&- \tfrac{1}{14000}\,[y\cdot[[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]\cdot x]] \\
&+ \tfrac{1}{14000}\,[[t_4(y,y,y,y)\cdot y]\cdot[[y\cdot y]\cdot[[y\cdot y]\cdot x]]] \\
&+ \tfrac{1}{14000}\,[[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[y\cdot y]]]\cdot[y\cdot x]] \\
&+ \tfrac{1}{14000}\,[y\cdot[[t_4(y,y,y^2,y^2)\cdot[y\cdot y]]\cdot x]] \\
&- \tfrac{1}{14000}\,[[t_4(y,y,y^2,y^2)\cdot[y\cdot y]]\cdot[y\cdot x]] \\
&+ \tfrac{1}{15750}\,[[y\cdot[t_4(y,y,y,yx)\cdot y]]\cdot[y\cdot[y\cdot y]]] \\
&- \tfrac{1}{16800}\,[[y\cdot y]\cdot[t_4(y,y,y,y)\cdot[[y\cdot y]\cdot[y\cdot x]]]] \\
&- \tfrac{1}{21000}\,[t_4(y,y,y,y)\cdot[y\cdot[[y\cdot y]\cdot[[y\cdot y]\cdot x]]]] \\
&- \tfrac{1}{21000}\,[y\cdot[t_4(y,y,y,yy^2)\cdot[[y\cdot y]\cdot x]]]
\end{aligned}
$$

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